Year 2019 Examination

ISC

Analysis of Pupil Performance

MATHEMATICS

Research Development and Consultancy Division

Council for the Indian School Certificate Examinations

New Delhi

Year 2019

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FOREWORD

This document of the Analysis of Pupils' Performance at the ISC Year 12 and ICSE Year 10 Examination is one of its kind. It has grown and evolved over the years to provide feedback to schools in terms of the strengths and weaknesses of the candidates in handling the examinations.

We commend the work of Mrs. Shilpi Gupta (Deputy Head) of the Research Development and Consultancy Division (RDCD) of the Council and her team, who have painstakingly prepared this analysis. We are grateful to the examiners who have contributed through their comments on the performance of the candidates under examination as well as for their suggestions to teachers and students for the effective transaction of the syllabus.

We hope the schools will find this document useful. We invite comments from schools on its utility and quality.

Gerry Arathoon
Chief Executive & Secretary

October 2019

PREFACE

The Council has been involved in the preparation of the ICSE and ISC Analysis of Pupil Performance documents since the year 1994. Over these years, these documents have facilitated the teaching-learning process by providing subject/ paper wise feedback to teachers regarding performance of students at the ICSE and ISC Examinations. With the aim of ensuring wider accessibility to all stakeholders, from the year 2014, the ICSE and the ISC documents have been made available on the Council's website www.cisce.org.

The documents include a detailed qualitative analysis of the performance of students in different subjects which comprises of examiners' comments on common errors made by candidates, topics found difficult or confusing, marking scheme for each question and suggestions for teachers/ candidates.

In addition to a detailed qualitative analysis, the Analysis of Pupil Performance documents for the Examination Year 2019 also have a component of a detailed quantitative analysis. For each subject dealt with in the document, both at the ICSE and the ISC levels, a detailed statistical analysis has been done, which has been presented in a simple user-friendly manner.

It is hoped that this document will not only enable teachers to understand how their students have performed with respect to other students who appeared for the ICSE/ISC Year 2019 Examinations, but also provide information on how they have performed within the Region or State, their performance as compared to other Regions or States, etc. It will also help develop a better understanding of the assessment/ evaluation process. This will help teachers in guiding their students more effectively and comprehensively so that students prepare for the ICSE/ ISC Examinations, with a better understanding of what is required from them.

The Analysis of Pupil Performance document for ICSE for the Examination Year 2019 covers the following subjects: English (English Language, Literature in English), Hindi, History, Civics and Geography (History and Civics, Geography), Mathematics, Science (Physics, Chemistry, Biology), Commercial Studies, Economics, Computer Applications, Economic Applications, Commercial Applications.

Subjects covered in the ISC Analysis of Pupil Performance document for the Year 2019 include English (English Language and Literature in English), Hindi, Elective English, Physics (Theory), Chemistry (Theory), Biology (Theory), Mathematics, Computer Science, History, Political Science, Geography, Sociology, Psychology, Economics, Commerce, Accounts and Business Studies.

I would like to acknowledge the contribution of all the ICSE and the ISC examiners who have been an integral part of this exercise, whose valuable inputs have helped put this document together.

I would also like to thank the RDCD team of Dr. M.K. Gandhi, Dr. Manika Sharma, Mrs. Roshni George and Mrs. Mansi Guleria who have done a commendable job in preparing this document.

Shilpi Gupta
Deputy Head - RDCD

October 2019

CONTENTS

INTRODUCTION

This document aims to provide a comprehensive picture of the performance of candidates in the subject. It comprises of two sections, which provide Quantitative and Qualitative analysis results in terms of performance of candidates in the subject for the ISC Year 2019 Examination. The details of the Quantitative and the Qualitative analysis are given below.

Quantitative Analysis

This section provides a detailed statistical analysis of the following:

- Overall Performance of candidates in the subject (Statistics at a Glance)
- State wise Performance of Candidates
- Gender wise comparison of Overall Performance
- Region wise comparison of Performance
- Comparison of Region wise performance on the basis of Gender
- Comparison of performance in different Mark Ranges and comparison on the basis of Gender for the top and bottom ranges
- Comparison of performance in different Grade categories and comparison on the basis of Gender for the top and bottom grades

The data has been presented in the form of means, frequencies and bar graphs.

Understanding the tables

Each of the comparison tables shows N (Number of candidates), Mean Marks obtained, Standard Errors and t-values with the level of significance. For t-test, mean values compared with their standard errors indicate whether an observed difference is likely to be a true difference or whether it has occurred by chance. The t-test has been applied using a confidence level of 95%, which means that if a difference is marked as ‘statistically significant’ (with * mark, refer to t-value column of the table), the probability of the difference occurring by chance is less than 5%. In other words, we are 95% confident that the difference between the two values is true.

t-test has been used to observe significant differences in the performance of boys and girls, gender wise differences within regions (North, East, South and West), gender wise differences within marks ranges (Top and bottom ranges) and gender wise differences within grades awarded (Grade 1 and Grade 9) at the ISC Year 2019 Examination.

The analysed data has been depicted in a simple and user-friendly manner.

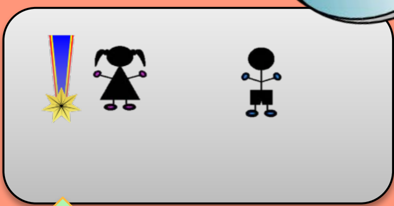
Given below is an example showing the comparison tables used in this section and the manner in which they should be interpreted.

Comparison on the basis of Gender

Gender	N	Mean	SE	t-value
Girls	2,538	66.1	0.29	11.91*
Boys	1,051	60.1	0.42	

*Significant at 0.05 level

Girls performed significantly better than boys.



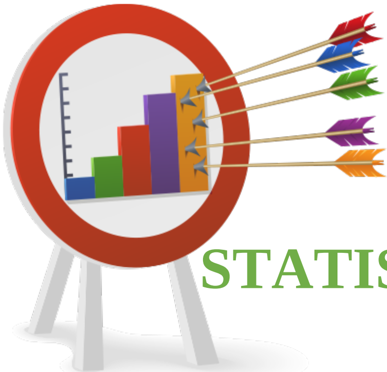
The results have also been depicted pictographically. In this case, the girls performed significantly better than the boys. This is depicted by the girl with a medal.

The table shows comparison between the performances of boys and girls in a particular subject. The t-value of 11.91 is significant at 0.05 level (mentioned below the table) with a mean of girls as 66.1 and that of boys as 60.1. It means that there is significant difference between the performance of boys and girls in the subject. The probability of this difference occurring by chance is less than 5%. The mean value of girls is higher than that of boys. It can be interpreted that girls are performing significantly better than boys.

Qualitative Analysis

The purpose of the qualitative analysis is to provide insights into how candidates have performed in individual questions set in the question paper. This section is based on inputs provided by examiners from examination centres across the country. It comprises of question wise feedback on the performance of candidates in the form of *Comments of Examiners* on the common errors made by candidates along with *Suggestions for Teachers* to rectify/ reduce these errors. The *Marking Scheme* for each question has also been provided to help teachers understand the criteria used for marking. Topics in the question paper that were generally found to be difficult or confusing by candidates, have also been listed down, along with general suggestions for candidates on how to prepare for the examination/ perform better in the examination.

QUANTITATIVE ANALYSIS



STATISTICS AT A GLANCE

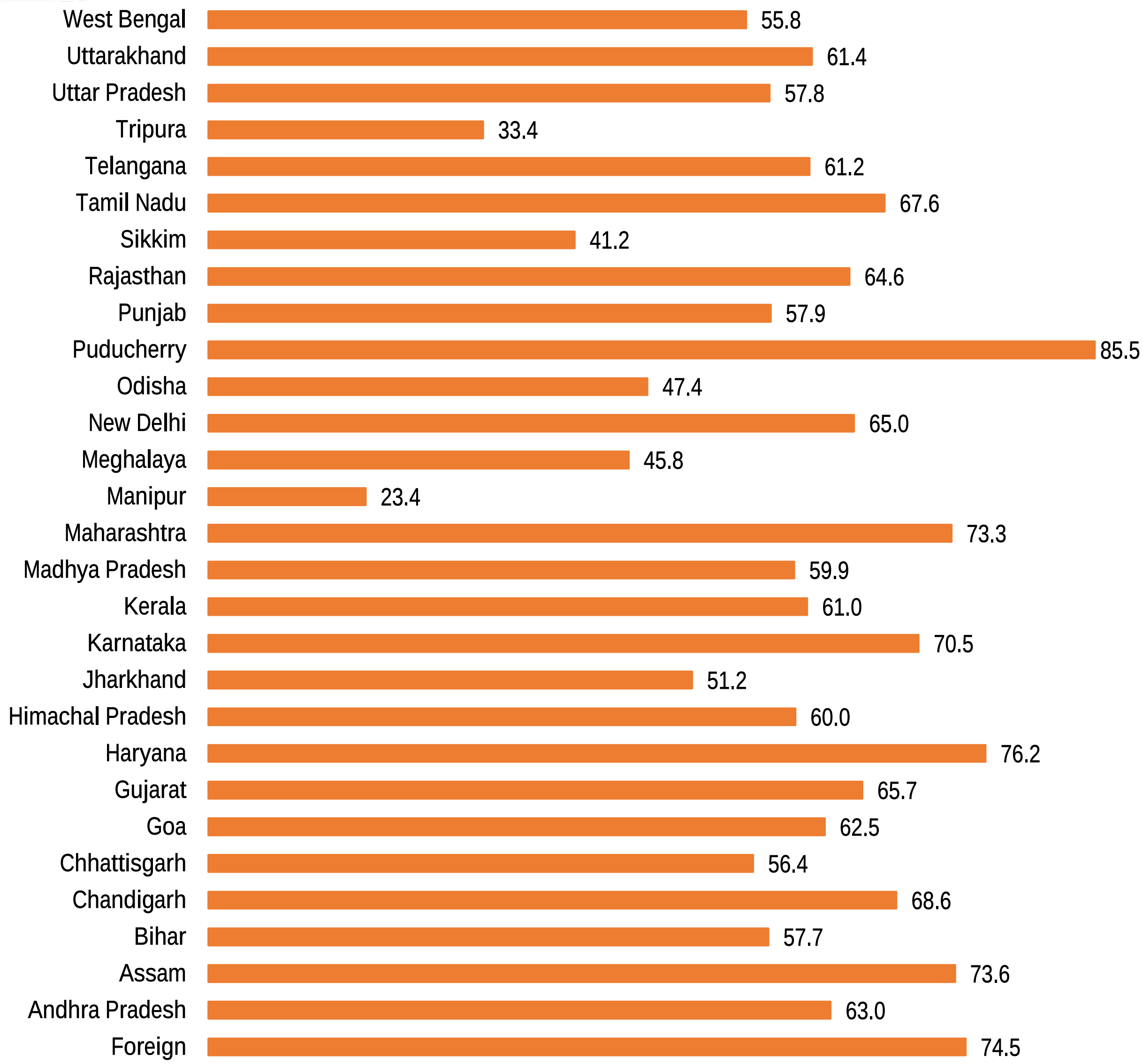
Total Number of Candidates: 49,950

Mean Marks: 58.3

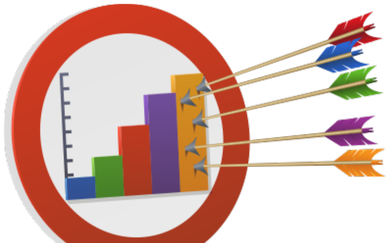
Highest Marks: 100

Lowest Marks: 0

PERFORMANCE (STATE-WISE & FOREIGN)



The States/ UTs of Puducherry, Haryana and Assam secured highest mean marks. Mean marks secured by candidates studying in schools abroad were 74.5.



GENDER-WISE COMPARISON

GIRLS

Mean Marks: 60.6

Number of Candidates: 19,288

BOYS

Mean Marks: 56.8

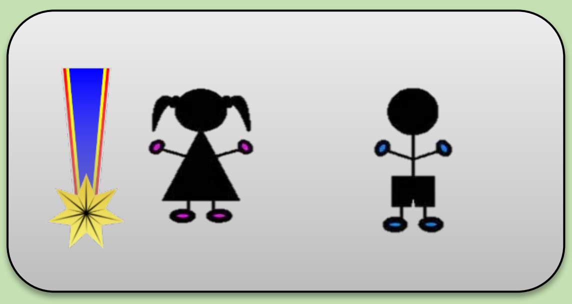
Number of Candidates: 30,662

Comparison on the basis of Gender

Gender	N	Mean	SE	t-value
Girls	19,288	60.6	0.17	16.59*
Boys	30,662	56.8	0.15	

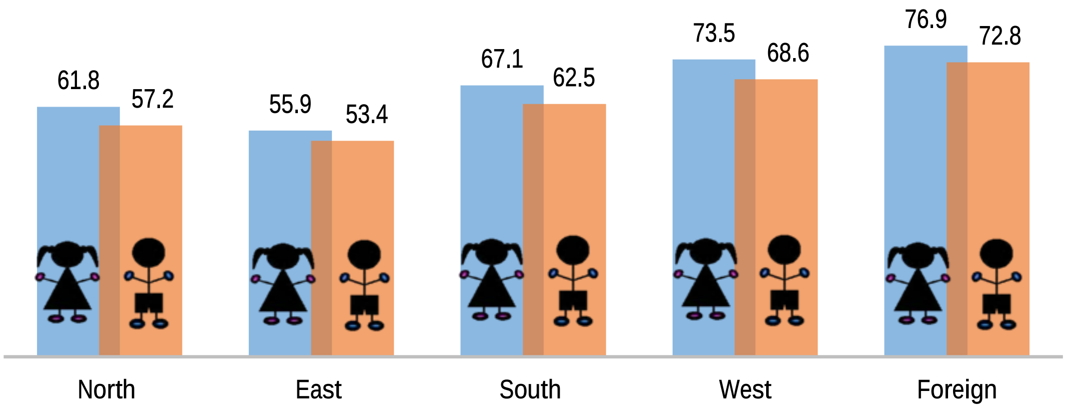
*Significant at 0.05 level

Girls performed significantly better than boys.



REGION-WISE COMPARISON

Mean Marks obtained by Boys and Girls-Region wise



Comparison on the basis of Gender within Region

Region	Gender	N	Mean	SE	t-value
North (N)	Girls	7,377	61.8	0.28	13.12*
	Boys	14,283	57.2	0.22	
East (E)	Girls	8,357	55.9	0.27	6.78*
	Boys	12,083	53.4	0.25	
South (S)	Girls	2,423	67.1	0.41	7.47*
	Boys	2,706	62.5	0.45	
West (W)	Girls	1,046	73.5	0.64	5.44*
	Boys	1,470	68.6	0.63	
Foreign (F)	Girls	85	76.9	2.04	1.39
	Boys	120	72.8	2.08	

*Significant at 0.05 level

The performance of girls was significantly better than that of boys in all the regions except foreign region, wherein no significant difference was observed between the performance of girls and boys.

REGION (N, E, S, W)

MARK RANGES : COMPARISON GENDER-WISE

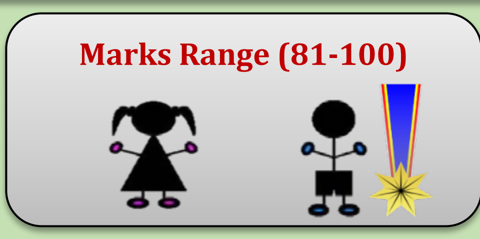
Comparison on the basis of gender in top and bottom mark ranges

Marks Range	Gender	N	Mean	SE	t-value
Top Range (81-100)	Girls	4,847	90.4	0.08	-4.31*
	Boys	7,490	90.8	0.06	
Bottom Range (0-20)	Girls	1,460	12.4	0.15	8.16*
	Boys	3,964	11.0	0.09	

*Significant at 0.05 level

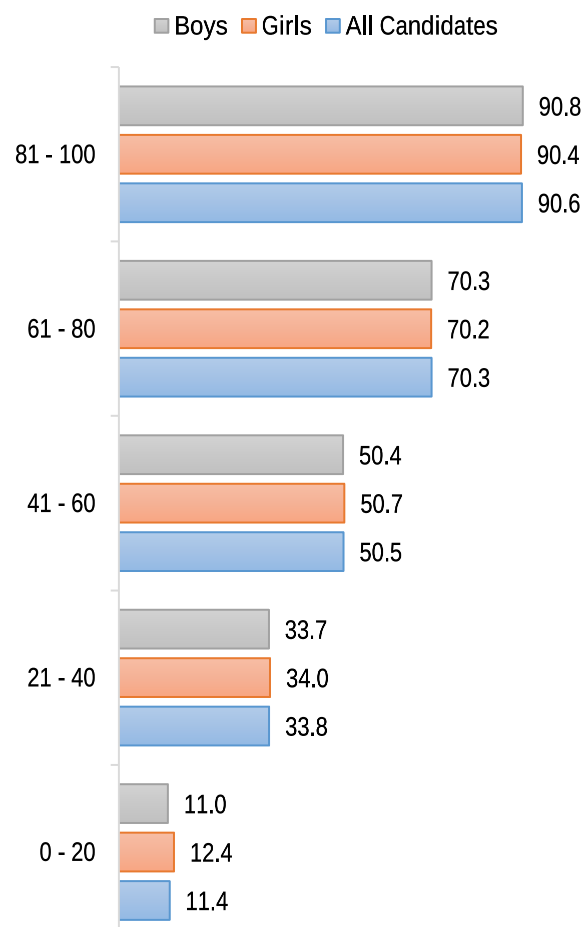
Marks Range (81-100)

Performance of boys was significantly better than the performance of girls.



Marks Range (0-20)

Performance of girls was significantly better than the performance of boys.



GRADES AWARDED : COMPARISON GENDER-WISE

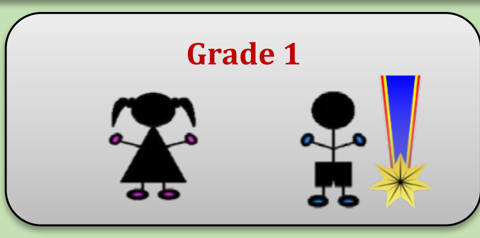
Comparison on the basis of gender in Grade 1 and Grade 9

Grades	Gender	N	Mean	SE	t-value
Grade 1	Girls	2,876	94.1	0.05	-3.19*
	Boys	4,650	94.3	0.04	
Grade 9	Girls	2,270	16.4	0.15	10.19*
	Boys	5,541	14.5	0.10	

*Significant at 0.05 level

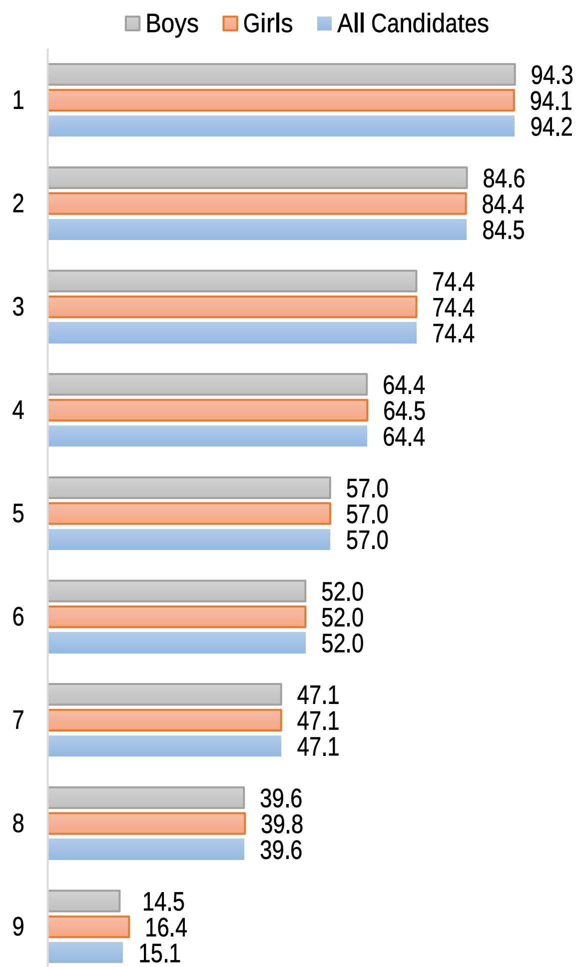
Grade 1

Performance of boys was significantly better than the performance of girls.



Grade 9

Performance of girls was significantly better than the performance of boys.



SECTION A (80 Marks)

Question 1

[10×2]

(i) If $f : R \rightarrow R$, $f(x) = x^3$ and $g : R \rightarrow R$, $g(x) = 2x^2+1$, and R is the set of real numbers, then find $fog\,(x)$ and $gof\,(x)$.

(ii) Solve: $\sin(2\tan^{-1}x) = 1$

(iii) Using determinants, find the values of k, if the area of triangle with vertices (−2, 0), (0, 4) and (0, k) is 4 square units.

(iv) Show that (A + A′) is symmetric matrix, if $A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$.

(v) $f(x) = \dfrac{x^2 - 9}{x - 3}$ is not defined at $x = 3$. What value should be assigned to $f(3)$ for continuity of $f(x)$ at $x = 3$?

(vi) Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on R.

(vii) Evaluate: $\displaystyle\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\,dx$

(viii) Using L'Hospital's Rule, evaluate: $\displaystyle\lim_{x \to 0} \frac{8^x - 4^x}{4x}$

(ix) Two balls are drawn from an urn containing 3 white, 5 red and 2 black balls, one by one without replacement. What is the probability that at least one ball is red?

(x) If events A and B are independent, such that $P(A) = \dfrac{3}{5}$, $P(B) = \dfrac{2}{3}$, find $P(A \cup B)$.

Comments of Examiners

(i) Majority of the candidates applied incorrect methods to solve the problem. Many candidates made errors in the process of simplification of algebraic functions after applying the concept.

(ii) Several candidates used incorrect conversion formula for 2 $\tan^{-1}x$ and thereby, were unable to complete the solution.

(iii) Many candidates wrote incorrect formula of the area of a triangle. Some candidates could not find the area of a triangle using determinant.

(iv) Most of the candidates calculated A + A′ but were unable to prove that it is a symmetric matrix.

(v) Majority of the candidates made errors in calculating limit of the given function.

(vi) Some candidates could not test the condition of increasing function after finding the derivative of the function.

(vii) Majority of the candidates simplified the function to $\tan^2x$ but couldn't complete the integration by taking correct identity of $\tan^2x$.

(viii) A large number of the candidates had an idea of solving limit of function by using the concept of L' Hospital's rule but had no knowledge of differentiating the function a^x.

(ix) Most of the candidates could find the probability of both red balls but failed to find the probability of one red ball and another non-red one.

(x) Several candidates failed to find the probability of mutually exclusive events and independent events.

Suggestions for teachers

- Explain to the students the concept of composite functions in detail.
- Illustrate conversion formulae for all inverse trigonometric functions with the help of diagrams.
- Clarify to the students, the application of determinant in finding the area of triangle and concept of collinearity when three points are given.
- Discuss the concept of symmetric matrix with ample examples.
- Elucidate the concept of continuity and differentiability.
- Emphasise on the need for revision of all trigonometric identities and formulae, differentiation of all types of standard functions, etc.
- Inculcate amongst the students, the habit of reading the questions carefully.
- Give enough practice to students in solving problems based on the probability concept when more than one ball is drawn at a time/one by one, with or without replacement.
- Emphasise on mutually exclusive events and independent events. A good number of problems need to be practiced by the students in order to get complete clarity on mutually exclusive events and independent events.
- Train students to solve questions by using proper logic and reasoning.

MARKING SCHEME

Question 1

(i)	$fog(x) = f[g(x)] = f(2x^2+1)$ $= (2x^2+1)^3$ $gof(x) = g[f(x)] = g(x^3)$ $= 2(x^3)^2 + 1 = 2x^6 + 1$
(ii)	Sin $(2\tan^{-1}x) = 1$ $\quad 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

	$\text{Sin}\left(\sin^{-1}\dfrac{2x}{1+x^2}\right)=1$ $\therefore \dfrac{2x}{1+x^2}=1 \Rightarrow x^2-2x+1=0$ $\Rightarrow (x-1)^2=0$ $\Rightarrow x=1$	2nd Solution: $\sin(2\tan^{-1}x)=1,$ $\sin(2\tan^{-1}x)=\sin\frac{\pi}{2}$ $\Longrightarrow 2\tan^{-1}x=\dfrac{\pi}{2}$ $\Longrightarrow \tan^{-1}x=\dfrac{\pi}{4}$ $\Longrightarrow x=1$
(iii)	Area of $\Delta=\frac{1}{2}\begin{vmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}$ $=\dfrac{1}{2}\begin{vmatrix}1 & -2 & 0\\ 1 & 0 & 4\\ 1 & 0 & k\end{vmatrix}=4$ $=\lvert 1(0)+2(k-4)+0\rvert=8$ $=\lvert k-4\rvert=4$ $\Rightarrow$k – 4 = $\pm$ 4 K = 8 or 0	
(iv)	A = $\begin{pmatrix}2 & 4\\ 3 & 5\end{pmatrix}$. A^1 = $\begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix}$ $A + A^1 = \begin{pmatrix}4 & 7\\ 7 & 10\end{pmatrix}$ $\left(A+A^1\right)^1 = \begin{pmatrix}4 & 7\\ 7 & 10\end{pmatrix}$ $\because \left(A+A^1\right)=\left(A+A^1\right)^1$ $\left(A+A^1\right)$ is symmetric matrix	
(v)	$\underset{x\to3}{lt}\ f(x)=\underset{x\to3}{lt}\dfrac{x^2-9}{x-3}=\underset{x\to3}{lt}\ 3+3=6$ $f(3)=6$	
(vi)	$f(x) = x^3 - 6x^2 + 12x + 5$ $f^1(x)=3x^2-12x+12$ $= 3\ (x^2 - 4x + 4)$ $= 3\ (x-2)^2 \geq 0\ \textit{for all}\ x\in R$ $\therefore f^1(x)\geq 0\ \ \forall x\in R$ $\therefore f(x)$ increasing function on R	

(vii)	$\int \frac{\sec^2 x}{\cos ec^2 x} dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \tan^2 x \ dx$ $= \int (\sec^2 x - 1) dx$ $= \tan x - x + c$
(viii)	$\underset{x \to 0}{lt} \frac{8^x - 4^x}{4x}$ $\frac{0}{0}$ Form Applying L Hospital's rule: $\underset{x \to 0}{lt} \frac{8^x \log 8 - 4^x \log 4}{4} = \frac{1}{4} \log \frac{8}{4}$ $= \frac{1}{4} \log 2$
(ix)	3W, 5R, 2B Event A: not drawing a red ball in 1st instance. Event B: not drawing a red ball in 2nd instance. P(A): $\frac{5}{10}$, $or \ \ P(B) = \frac{4}{9}$ Probability of not drawing red ball in 1st and 2nd instances: = P(A). P(B) = $\frac{5}{10} \cdot \frac{4}{9}$ $\therefore$ probability of drawing atleast one red ball is = 1-P(A).P(B) $= 1 - \frac{5}{10} \cdot \frac{4}{9}$ $= 1 - \frac{2}{9} = \frac{7}{9}$ = 0.777 = 0.78
(x)	$P(A \cup B) = P(A) + (B) - P(A \cap B)$ $P(A \cap B) = P(A).P(B)$ $= \frac{3}{5} \cdot \frac{2}{3} = \frac{2}{5}$ $P(A \cup B) = \frac{3}{5} + \frac{2}{3} - \frac{2}{5} = \frac{13}{15}$ = 0.8666

Question 2

[4]

If $f : A \to A$ and A = R – $\left\{ 8/5 \right\}$, show that the function *f(x)* = $\frac{8x+3}{5x-8}$ is one – one onto.

Hence, find f^{-1}.

Comments of Examiners

Most of the candidates proved that the given function is *one-one* function, but some candidates could not give any justification to say that it is *onto* function also.

Suggestions for teachers

Illustrate with relevant examples that the function, Co-domain and Range are always equal for onto function.

MARKING SCHEME

Question 2

Let $x_1, x_2 \in$ A such that $f(x_1) = f(x_2)$

$\therefore \frac{8x_1+3}{5x_1-8} = \frac{8x_2+3}{5x_2-8} \Rightarrow 40x_1x_2 - 64x_1 + 15x_2 - 24$

$= 40x_1x_2 + 15x_1 - 64x_2 = 24$

$\Rightarrow 79x_1 = 79x_2$

$\therefore x_1 = x_2$

$\therefore$ the function f is one one.

Let $y = f(x)$ $\quad y = \frac{8x+3}{5x-8}$

$5xy - 8y = 8x + 3$

$5xy - 8x = 8y + 3$

$x(5y - 8) = 8y + 3$

$x = \frac{8y+3}{5y-8}$ when $y \in$ A

$\Rightarrow 5y - 8 \neq 0$

$\Rightarrow y \neq 8/5$

$\therefore$ Range of f = R $- \left\{\frac{8}{5}\right\}$ = A

$\Rightarrow f$ is onto

$\therefore$ the function of f is one one onto $\Rightarrow$ f is invertible.

$x = \frac{8y+3}{5y-8}$

$f^{-1}(y) = \frac{8y+3}{5y-8}$ *i.e.* $f^{-1}(x) = \frac{8x+3}{5x-8}$

Question 3

[4]

(a) Solve for x:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

OR

(b) If $\sec^{-1} x = \mathrm{cosec}^{-1} y$, show that $\frac{1}{x^2} + \frac{1}{y^2} = 1$

Comments of Examiners

(a) A few candidates not only wrote an incorrect formula for $\tan^{-1}x + \tan^{-1}y$ but also made errors while simplifying the expression of LHS.

(b) Several candidates made errors while converting the functions $\text{Sec}^{-1}x$ and $\text{Cosec}^{-1}x$ to its correct form. Some candidates could not complete the solution.

Suggestions for teachers

- Explain the formulae of all inverse trigonometric functions for sum /difference of two or more terms and give adequate practice in solving questions to prove the required result.
- Train students to convert formulae for all inverse trigonometric functions with the help of diagrams and give adequate practice in solving questions based on the different types of conversions of inverse circular functions.

MARKING SCHEME

Question 3

(a)

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}.\frac{x+1}{x+2}}\right)=\frac{\pi}{4}$$

$$\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}=1$$

$$2x^2=1$$

$$x^2=\frac{1}{2}$$

$$x=\pm\frac{1}{\sqrt{2}}$$

OR

(b)

$$\sec^{-1}x=\cos ec^{-1}y \Rightarrow \cos^{-1}{}^{1}\!/_{x}=\sin^{-1}{}^{1}\!/_{y}=\theta$$

$$\cos^{-1}{}^{1}\!/_{x}=\theta \quad \sin^{-1}{}^{1}\!/_{y}=\theta$$

$$\cos\theta=\frac{1}{x} \quad \sin\theta={}^{1}\!/_{y}$$

$$\Rightarrow \cos^2\theta+\sin^2\theta=\frac{1}{x^2}+\frac{1}{y^2}$$

$$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}=1$$

Question 4

[4]

Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

Comments of Examiners

Several candidates did not apply the properties of determinants in the correct order to reduce further and did not take common factors in a row/column. Some candidates solved this problem by expanding the determinant directly.

Suggestions for teachers

- Explain properties and applications of determinants with the help of relevant examples.
- Give adequate practice in solving questions based on different types of properties.
- Emphasise on revision of basic concepts of Algebra.

MARKING SCHEME

Question 4

$$\begin{vmatrix} x & x(x^2+1) & x+1 \\ y & y(y^2+1) & y+1 \\ z & z(z^2+1) & z+1 \end{vmatrix} = \begin{vmatrix} x & x(x^2+1) & x \\ y & y(y^2+1) & y \\ z & z(z^2+1) & z \end{vmatrix} + \begin{vmatrix} x & x(x^2+1) & 1 \\ y & y(y^2+1) & 1 \\ z & z(z^2+1) & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} + \begin{vmatrix} x & x & 1 \\ y & y & 1 \\ z & z & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$c_2 : c_2 - c_1$ and $c_3 : c_3 - c_1$

$$\begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

R_1: $R_1 - R_2$ and R_2: $R_2 - R_3$

$$\begin{vmatrix} x-y & x^3-y^3 & 0 \\ y-z & y^3-z^3 & 0 \\ z & z^3 & 1 \end{vmatrix} = (x-y)\ (y-z) \begin{vmatrix} 1 & x^2+y^2+xy & 0 \\ 1 & y^2+z^2+yz & 0 \\ z & z^3 & 1 \end{vmatrix}$$

Again R_2: $R_2 - R_1$

$$(x-y)(y-z)\begin{vmatrix} 1 & x^2+y^2+xy & 0 \\ 0 & z^2-x^2+yz-xy & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$(x-y)(y-z)\begin{vmatrix} 1 & x^2+y^2+xy & 0 \\ 0 & (z-x)(z+x)+y(z-x) & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$(x-y)(y-z)(z-x)(x+y+z)\begin{vmatrix} 1 & x^2+y^2+xy & 0 \\ 0 & 1 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$\therefore \Rightarrow (x-y)(y-z)(z-x)(x+y+z)$$

Question 5

[4]

(a) Show that the function $f(x) = |x-4|,\ x \in R$ is continuous, but not differentiable at $x = 4$.

OR

(b) Verify the Lagrange's mean value theorem for the function:

$f(x) = x + \frac{1}{x}$ in the interval [1, 3]

Comments of Examiners

(a) Most of the candidates attempted the first part i.e. proving continuity of a function but did not apply conditions for differentiability to examine the same.

(b) Several candidates made errors while applying properties of mean value theorem due to lack of understanding of open and closed intervals. A few candidates made calculation errors.

Suggestions for teachers

- Explain to the students, the concept of continuity and differentiability with the help of suitable examples.
- Discuss open and closed intervals and their significance with the help of examples in the class.
- Clarify that continuity is in closed interval [,] and differentiability is in open interval (,).

MARKING SCHEME

Question 5

(a)

$f(x) = |x-4| \ \forall\, x \in \mathrm{R}$

$$f(x) = \begin{cases} x-4 & x>4 \\ 4-x & x<4 \\ 0 & x=4 \end{cases}$$

$$\text{LHL} = \underset{x\to 4^-}{lt}\ f(x)$$

$$= \underset{h\to 0}{lt}\ f(4-h) = \underset{h\to 0}{lt}\ 4-(4-h) = 0$$

RHL = $\underset{x\to 4^+}{lt} f(x) = \underset{h\to 0}{lt} f(4+h) = \underset{h\to 0}{lt}(4+h)-4=0$

$f(4) = 0$

$f(4)$ =LHL = RHL $\therefore f(x)$ is continuous at $x = 4$

LHD = $\underset{x\to 4^-}{lt} f^1(x) = \underset{h\to 0}{lt} \dfrac{f(4-h)-f(4)}{-h}$

$= \lim\limits_{h\to 0} \dfrac{[4-(4-h)]-0}{-h} = -1$

RHD = $\underset{x\to 4^-}{lt} f^1(x) = \underset{h\to 0}{lt} \dfrac{f(4+h)-f(4)}{h}$

$= \underset{h\to 0}{lt} \dfrac{(4+h)-4-0}{h} = 1$

LHD $\neq$ RHD

$\therefore f$ is not differentiable at $x = 4$.

OR

(b)

$f(x) = x + \dfrac{1}{x}$ is continuous in the closed interval $1 \le x \le 3$ i.e [1,3]

$f^1(x) = 1 - \dfrac{1}{x^2}$ is existing in the open interval $1 < x < 3$. i.e (1, 3)

The conditions of Lagrange's Mean Value Theorem are satisfied.

$f(1) = 1+1 = 2 \quad f(3) = 3 + \dfrac{1}{3} = \dfrac{10}{3}$

To verify further, need to show that there exists a 'c' $\in$ (1, 3) such that

$f^1(c) = \dfrac{f(b)-f(a)}{b-a}$

$1 - \dfrac{1}{x^2} = \dfrac{10/3 - 2}{3-1} = \dfrac{4}{3} \times \dfrac{1}{2} = \dfrac{2}{3}$

$1 - \dfrac{1}{x^2} = \dfrac{2}{3}$

$\dfrac{1}{x^2} = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

$x^2 = 3$

$x = \pm\sqrt{3} = +1.73$ lies in the open interval (1, 3)

Question 6

[4]

If $y = e^{\sin^{-1}x}$ and $z = e^{-\cos^{-1}x}$, prove that $\dfrac{dy}{dz} = e^{\pi/2}$

Comments of Examiners

Some common errors made by many candidates in this question were:

- differentiation of exponential functions
- elimination of the parameter
- errors in simplification.

Suggestions for teachers

- Explain the concept of derivatives of exponential functions and parametric functions with ample examples.
- Give enough practice to students in solving different types of problems on the topic.

MARKING SCHEME

Question 6

$$\frac{dy}{dx} = e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}, \quad \frac{dz}{dx} = e^{-\cos^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{e^{\sin^{-1}x} \cdot 1/\sqrt{1-x^2}}{e^{-\cos^{-1}} x. \ 1/\sqrt{1-x^2}}$$

$$= e^{\sin^{-1}x + \cos^{-1}x} = e^{\pi/2}$$

Question 7

[4]

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level. The bottom of the ladder is pulled away from the wall, along the ground, at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

Comments of Examiners

Several candidates could not form the function. Many candidates made errors while differentiating the function with reference to time and ended up differentiating with respect to 'x' instead of time 't'.

Suggestions for teachers

Explain the concepts of all types of applications of derivatives and give ample practice on the topic.

MARKING SCHEME

Question 7

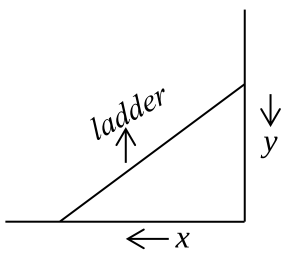
$$x^2 + y^2 = 169$$

$$at \ x = 5 \quad y^2 = 169 - 25 = 144$$

$$y = 12$$

$$\frac{dx}{dt} = 2m/s$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$x=5, y=12 \text{and} \frac{dx}{dt}=2m/s$$

$$5\times 2+12.\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{5}{6}m/s$$

Question 8

[4]

(a) Evaluate: $\int \frac{x(1+x^2)}{1+x^4}dx$

OR

(b) Evaluate: $\int_{-6}^{3} |x+3|\, dx$

Comments of Examiners

(a) Majority of the candidates did not split the given function into two individual functions. Many candidates applied incorrect substitution and couldn't complete the solution in the right manner.

(b) Many candidates made errors while splitting the given interval into two parts. Some candidates made calculation errors even after applying correct limits.

Suggestions for teachers

- Teach the concept - integration by substitution by giving variety of problems in the class and ensure that the students also get enough practice in solving problems based on this concept on their own.
- Explain the concept of splitting the given interval into two or more intervals in case of absolute functions and properties of definite integrals.

MARKING SCHEME

Question 8

(a)

$$\int \frac{x(1+x^2)}{1+x^4}dx \qquad x^2=t \qquad dt=2x\,dx$$

$$\frac{1}{2}\int \frac{1+t}{1+t^2}dt$$

$$=\frac{1}{2}\int \frac{1}{1+t^2}dt+\frac{1}{4}\int \frac{2t}{1+t^2}dt$$

$$=\frac{1}{2}\tan^{-1}t+\frac{1}{4}\log(1+t^2)+C$$

$$=\frac{1}{2}\tan^{-1}x^2+\frac{1}{4}\log(1+x^4)+C$$

	OR
(b)	$\int_{-6}^{3} \lvert x+3 \rvert dx$ $= \int_{-6}^{-3} -(x+3)dx + \int_{-3}^{3} (x+3)dx$ \ $= -\left(\frac{x^2}{2}+3x\right)_{-6}^{-3} + \left(\frac{x^2}{2}+3x\right)_{-3}^{3} = \frac{9}{2}+18 = \frac{45}{2}$ $x+3>0 \Rightarrow x>-3$ $x+3<0 \Rightarrow x<-3$ $\Rightarrow -(x+3)>0$

Question 9

[4]

Solve the differential equation: $\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1}$

Comments of Examiners

Majority of the candidates identified the given differential equation as homogeneous equation whereas, it was the type of equation reducible to simple separable.

Suggestions for teachers

- Explain all types of differential equations and applications and give adequate practice on various types of problems.
- Clarify the significance of constant in the solution of a differential equation.

MARKING SCHEME

Question 9

$$\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1}$$

Let $x + y = z$

$$1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dZ}{dx} = \frac{Z+2}{2Z-1} + 1$$

$$\frac{dz}{dx} = \frac{z+2+2z-1}{2z-1}$$

$$\frac{dz}{dx} = \frac{3z+1}{2z-1}$$

$$\int \frac{2z-1}{3z+1} dz = \int dx + c \Rightarrow \frac{2}{3}\left[z - \frac{5}{6}\log(3z+1)\right] = x + c$$

$$\frac{2}{3}(x+y) - \frac{5}{9}\log[3x+3y+1] = x + c$$

Question 10

[4]

Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two balls are drawn from Bag A and placed in Bag B. Then, what is the probability of drawing a white ball from Bag B?

Comments of Examiners

A large number of candidates could not identify all possible cases while transferring two balls from Bag A to Bag B and drawing one white ball from Bag B. Candidates also made errors while writing the summation of product of probabilities of all the cases.

Suggestions for teachers

- Explain concepts related to the theory of probability and make the students practice problems on reasoning out all possible cases in a given random experiment.
- Illustrate, with examples, cases in which there is a need to combine the addition rule and product rule (Summation of products).

MARKING SCHEME

Question 10

Bag A	Bag B
4 W	3 W
3 B	5 B

Two balls are drawn from Bag A.

∴ 2W or 2B or 1W & 1B.

$\frac{4C_2}{7C_2}$ $\quad\frac{3C_2}{7C_2}$ $\quad\frac{4C_1 \times 3C_1}{7C_2}$

Bag B	5W 5B	3W 7B	4W 6B
	$\frac{5}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Probability of drawing a white ball from bag B $= \frac{4C_2}{7C_2} \times \frac{5}{10} + \frac{3C_2}{7C_2} \times \frac{3}{10} + \frac{4C_1 \times 3C_1}{7C_2} \times \frac{4}{10}$

$= \frac{1}{7} + \frac{3}{70} + \frac{16}{70} = \frac{29}{70}$

Question 11

[6]

Solve the following system of linear equations using matrix method:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9$$

$$\frac{2}{x} + \frac{5}{y} + \frac{7}{z} = 52$$

$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

Comments of Examiners

Some candidates made simplification errors in the process of finding the values of variables *x*, *y* and *z*. A few candidates made errors while calculating cofactors and inverse of a matrix.

Suggestions for teachers

Ensure that students are given adequate practice in computing cofactors, inverse of a given matrix and in finding solution of a system of equations in the matrix form.

MARKING SCHEME

Question 11

The equations are

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$$

$$\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$$

$$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$$

$$\frac{1}{x}=a,\frac{1}{y}=b,\frac{1}{z}=c$$

Equation are $a+b+c=9$

$2a+5b+7c=52$

$2a+b-c=0$

Matrix equation $A=\begin{bmatrix}1&1&1\\2&5&7\\2&1&-1\end{bmatrix},B=\begin{bmatrix}9\\52\\0\end{bmatrix},X=\begin{bmatrix}a\\b\\c\end{bmatrix}$

$$Ax=B$$

$$|A|=-4\neq 0$$

$$X=A^{-1}B$$

$$A^{-1}=-\frac{1}{4}\begin{bmatrix}-12&2&2\\16&-3&-5\\-8&1&3\end{bmatrix}$$

$$\begin{bmatrix}a\\b\\c\end{bmatrix}=-\frac{1}{4}\begin{bmatrix}-12&2&2\\16&-3&-5\\-8&1&3\end{bmatrix}\begin{bmatrix}9\\52\\0\end{bmatrix}=-\frac{1}{4}\begin{bmatrix}-4\\-12\\-20\end{bmatrix}=\begin{bmatrix}1\\3\\5\end{bmatrix}$$

$$a=1,b=3,c=5\Rightarrow x=1,y=\frac{1}{3},z=\frac{1}{5}$$

Question 12 [6]

(a) The volume of a closed rectangular metal box with a square base is 4096 cm^3. The cost of polishing the outer surface of the box is ₹ 4 per cm^2. Find the dimensions of the box for the minimum cost of polishing it.

OR

(b) Find the point on the straight line $2x + 3y = 6$, which is closest to the origin.

Comments of Examiners

(a) Several candidates failed to apply the correct formula for the surface area of a closed box with square base and express the same in terms of one variable. Some could not complete the 2nd order derivative test for maximization/minimization. A few candidates found the value of the variable but could not find the dimensions of the box.

(b) A number of candidates identified distance function in terms of one variable in the square root form where differentiation of it involves long calculations. Some candidates could not complete the second order derivative test.

Suggestions for teachers

- Ensure proper revision of mensuration formulae of 2-dimensional and 3-dimensional figures/ objects.
- Explain the method for identifying the objective function, re-writing in terms of one variable and applying the concept of maxima/minima (1st derivative test and 2nd derivative test).
- Illustrate the method of forming the function in terms of one variable in such a manner that students understand it easily. Tell students about eliminating the square root of a function by squaring on both sides before differentiating.

MARKING SCHEME

Question 12

(a) Let the side of a square base= x

Height = h

$x^2h = 4096$

or $h = \dfrac{4096}{x^2}$

$T.S.A.(S) = 2x^2 + 4xh$

$S = 2x^2 + 4x \times \dfrac{4096}{x^2}$

$S = 2x^2 + \dfrac{16384}{x}$

$=$ Cost of polishing $(c) = 4\left(2x^2 + \dfrac{16384}{x}\right)$

$\dfrac{dc}{dx} = 4\left[4x - \dfrac{16384}{x^2}\right]$

$\dfrac{dc}{dx} = 0 \Rightarrow 4x = \dfrac{16384}{x^2}$

$x^3 = 4096$

$x = 16$

	$\frac{d^2c}{dx^2} = 4\left[4 + \frac{32768}{x^3}\right]$ at $x = 16$, $\frac{d^2c}{dx^2} = \text{positive}$ therefore the cost is minimum at x= 16 hence, $\text{h} = \frac{4096}{16^2} = 16$
OR	
(b)	Straight line: $2x + 3y = 6$ Let the point be (x, y) $\Rightarrow y = \frac{6-2x}{3}$ The shortest distance is the perpendicular distance $d = \sqrt{x^2 + y^2}$ or $D = x^2 + y^2$ $D = x^2 + y^2$ $D = x^2 + \left(\frac{6-2x}{3}\right)^2$ $\frac{dD}{dx} = 2x + 2\left(\frac{6-2x}{3}\right) \cdot \frac{-2}{3} = \frac{26x - 24}{9}$ with $\frac{dD}{dx} = 0$ $x = \frac{12}{13}$ $\frac{d^2D}{dx} = \frac{26}{9}[> 0]$ Therefore, the distance is minimum at $x = \frac{12}{13}$ So $y = \frac{18}{13}$ $\left(\frac{12}{13}, \frac{18}{13}\right)$ is the required point on the line.

Question 13

[6]

Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

Comments of Examiners

Most candidates attempted this problem to the extent of applying properties of definite integrals and found the value of 2I. However, they could not apply the correct method to find the integral of the function and hence were unable to simplify further.

Suggestions for teachers

- Explain, with examples, the properties of integrals and applications on them at different difficulty levels.
- Give adequate practice to the students in solving different types of problems.

MARKING SCHEME

Question 13

$$I = \int_0^{\pi} \frac{x \tan x}{Sec\, x + \tan x}\, dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{Sec(\pi - x) + \tan(\pi - x)} = \pi \int_0^{\pi} \frac{\tan x}{Sec\, x + \tan x} - I$$

$$2I = \pi \int_0^{\pi} \frac{\tan x}{Sec\, x + \tan x} \times \frac{Sec\, x - \tan x}{Sec\, x + \tan x} dx$$

$$2I = \pi \int_0^{\pi} (\tan x\ Sec\, x - Sec^2 x + 1)\ dx$$

$$2I = \pi[Sec\, x - \tan x + x]_0^{\pi}$$

$$2I = \pi[(-1 + \pi) - (1 - 0)]$$

$$2I = \pi[\pi - 2]$$

$$I = \pi \frac{(\pi - 2)}{2}$$

Question 14

[6]

(a) Given three identical Boxes A, B and C, Box A contains 2 gold and 1 silver coin, Box B contains 1 gold and 2 silver coins and Box C contains 3 silver coins. A person chooses a Box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver.

OR

(b) Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$.

Also, find the probability of obtaining at most one success.

Comments of Examiners

(a) Most of the candidates attempted this question well. However, some candidates could not identify the Box C, from where a silver coin was drawn. A few candidates, instead of using Bayes' theorem, applied the concept of total probability.

(b) Many candidates used incorrect formula for Standard Deviation and calculated incorrect values of n, p and q. Some candidates could not understand the meaning of *at most one success*.

Suggestions for teachers

- Teach Bayes' theorem as one of the concepts of conditional probability with illustrations and tree diagram.
- Explain the concept and all the formulae of Binomial probability distribution and representing as B (n, p) of $(q+p)^n$.
- Give adequate practice to students on various types of problems.

MARKING SCHEME

Question 14

(a)

Boxes	A	B	C
	2 G 1S	1 G 2 S	3 S

$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$

(Drawing a silver coin
The box chosen is Box C)

$$P\left(\frac{A}{E_1}\right) = \frac{1}{3}, P\left(\frac{A}{E_2}\right) = \frac{2}{3}, P\left(\frac{A}{E_3}\right) = 1$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times 1} = \frac{1}{\frac{1}{3} + \frac{2}{3} + 1} = \frac{1}{2}$$

OR

(b)

$$np = 9, \sqrt{npq} = \frac{3}{2}$$

$$npq = \frac{9}{4}$$

$$9q = \frac{9}{4}$$

$$q = \frac{1}{4}, p = \frac{3}{4}$$

$$n \times \frac{1}{\cancel{4}} \times \frac{3}{4} = \frac{9}{\cancel{4}}$$

$$n = 12$$

Binomial distribution $-\left(\frac{1}{4} + \frac{3}{4}\right)^{12}$

For at most one success

$P(X = 0) + P(X = 1)$

$12C_0\, q^{12} P^0 + {}^{12}C_1 q^{11} P^1$

$$1 \times \left(\frac{1}{4}\right)^{12} + 12\left(\frac{1}{4}\right)^{11}\left(\frac{3}{4}\right)$$

$$\left(\frac{1}{4}\right)^{12}[1 + 36]$$

$$\left(\frac{1}{4}\right)^{12} \times 37$$

SECTION B (20 Marks)

Question 15

[3×2]

(a) If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}| = 13$ and $|\vec{a}| = 5$, find the value of $|\vec{b}|$.

(b) Find the length of the perpendicular from origin to the plane $\vec{r}.(3i - 4j - 12k) + 39 = 0$.

(c) Find the angle between the two lines $2x = 3y = -z$ and $6x = -y = -4z$.

Comments of Examiners

(a) Majority of the candidates performed well. However, a few candidates applied incorrect formula for perpendicular vectors.

(b) Many candidates did not apply the perpendicular distance formula from a given point to a plane. Some candidates attempted to express it in the normal form but made calculation errors.

(c) Most candidates attempted well barring a few who could not express the given equation into the desired form and hence, could not calculate the direction ratios of the lines correctly.

Suggestions for teachers

- Clarify the concepts of dot product and cross product of two vectors and their properties.
- Explain the concept of converting cartesian form of equations of plane / straight line to vector form and vice versa and formulae for perpendicular distance with the help of illustrations.
- Teach in detail the method of identifying direction ratios and conversion of vector form to symmetric form and *vice versa*.

MARKING SCHEME

Question 15

(a)	$\vec{a} \cdot \vec{b} = 0$ since they are perpendicular $|\vec{a}+\vec{b}| = 13 \qquad |a| = 5$ $|\vec{a}+\vec{b}|^2 = 169$ $(\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b}) = 169$ $|a|^2 + |b|^2 + 2\vec{a}.\vec{b} = 169$ $25 + |b|^2 + 2 \times 0 = 169$ $|\vec{b}|^2 = 144$ $|\vec{b}| = 12$
(b)	The equation may be written as $3x - 4y - 12z + 39 = 0$ Distance from origin=$\frac{39}{\sqrt{3^2+(-4)^2+(-12)^2}} = \frac{39}{\sqrt{9+16+144}} = \frac{39}{13} = 3$ units

(c)	Equations are $2x = 3y = -z, \quad 6x = -y = -4z$ $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ $(3,2,-6)$ and $(2,-12,-3)$ *are direction cosines of the lines.* $Cos\,\theta = \frac{6-24+18}{\sqrt{9+4+36}\sqrt{4+144+9}}$ $Cos\,\theta = \frac{0}{7 \times \sqrt{157}}$ $Cos\,\theta = 0$ $\theta = 90°$

Question 16

[4]

(a) If $\vec{a} = i - 2j + 3k$, $\vec{b} = 2i + 3j - 5k$, prove that $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular.

OR

(b) If $\vec{a}$ and $\vec{b}$ are non-collinear vectors find the value of x such that the vectors $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.

Comments of Examiners

(a) Majority of the candidates calculated a × b but failed to apply the concept of scalar triple product a. (a × b) =0. A few candidates made calculation errors while finding a × b.

(b) Many candidates had no idea of the concept of the collinear vectors and coplanar vectors. Some candidates made errors while comparing the components of collinear vectors.

Suggestions for teachers

- Explain the concept of scalar triple product and its applications with the help of examples.
- Clarify the concepts like collinear vectors, coplanar vectors and linear combination of vectors, etc.
- Give adequate practice to the students in solving problems based on above concepts.

MARKING SCHEME

Question 16

(a)	$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$ $\vec{a} \times \vec{b} = i + 11j + 7k$ $\vec{a} \cdot (\vec{a} \times \vec{b})$ $(i - 2j + 3k) \cdot (i + 11j + 7k)$ $1 - 22 + 21 = 0$ **OR**
(b)	Since $\vec{\alpha}$ & $\vec{\beta}$ are collinear

	$\vec{\alpha} = k\vec{\beta}$ $(x-2)\vec{a} + \vec{b} = k\left[(3+2x)\vec{a} - 2\vec{b}\right]$ $x - 2 = k(3+2x)$ $1 = -2k, k = -\frac{1}{2}$ $x - 2 = -\frac{1}{2}(3+2x)$ $2x - 4 = -3 - 2x$ $4x = 1$ $x = \frac{1}{4}$

Question 17

[4]

(a) Find the equation of the plane passing through the intersection of the planes $2x + 2y - 3z - 7 = 0$ and $2x + 5y + 3z - 9 = 0$ such that the intercepts made by the resulting plane on the x-axis and the z-axis are equal.

OR

(b) Find the equation of the lines passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

Comments of Examiners

(a) Majority of the candidates could not find the correct values of x-intercept and z-intercept.

(b) Most candidates did not apply the concept of perpendicular lines and condition for the same correctly. Some candidates made calculation errors while applying cross multiplication rule to find direction ratios.

Suggestions for teachers

- Explain the concepts of family of planes and all possible conditions that may be involved in problems on this topic.
- Clarify the different methods of finding the equation of straight line by applying different types of conditions.

MARKING SCHEME

Question 17

(a)	The required plane will be: $2x + 2y - 3z - 7 + k(2x + 5y + 3z - 9) = 0$ $(2 + 2k)x + (2 + 5k)y + (3k - 3)z = 7 + 9k$ since x intercept $= z$ intercept $3k - 3 = 2 + 2k$ $k = 5$ and $(2 + 10)x + (2 + 25)y + 12z = 52$ $12x + 27y + 12z = 52$ **OR**

(b)	Let the direction ratios of the line be *a*, *b*, *c* and the direction ratios of two given lines are< 1, 2, 3 > and < -3, 2, 5> Since the line is perpendicular to the other two lines therefore $a+2b+3c=0$ $-3a+2b+5c=0$ $\frac{a}{10-6}=\frac{b}{-9-5}=\frac{c}{2+6} \Rightarrow \frac{a}{4}=\frac{b}{-14}=\frac{c}{8}$ $\frac{a}{2}=\frac{b}{-7}=\frac{c}{4}$ Direction ratios of required line are < 2, -7, 4 > And the line passes through the point (2, 1,3) Hence the equation will be: $\frac{x-2}{2}=\frac{y-1}{-7}=\frac{z-3}{4}$

Question 18

[6]

Draw a rough sketch and find the area bounded by the curve $x^2 = y$ and $x + y = 2$.

Comments of Examiners

A number of candidates attempted this question well. Some candidates made mistakes in sketching the two curves. Hence, they could not find the common portion and limits correctly.

Suggestions for teachers

- Give practice in sketching different types of curves and interpretation of the graphs.
- Explain the method of identifying the limits of the definite integral and area bounded by the region for the given curves and X-axis, etc.

MARKING SCHEME

Question 18

Curve $y = x^2$

Line $x + y = 2$

points of intersection

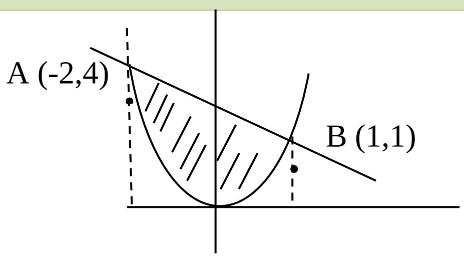
$x + x^2 = 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2,\ x = 1 \qquad A(-2,4)$

$y = 4,\ y = 1 \qquad B(1,1)$



Area of the shaded region

$$\underset{[Line]}{\int_{-2}^{1} y\,dx} - \underset{[Curve]}{\int_{-2}^{1} y\,dx} = \int_{-2}^{1} (2-x)\ dx - \int_{-2}^{1} x^2 dx$$

$$\left[\left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\right]_{-2}^{1} \quad \text{(values)}$$

$$\left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - \frac{4}{2} + \frac{8}{3}\right)$$

$$2 - \frac{1}{2} - \frac{1}{3} + 4 + 2 - \frac{8}{3}$$

$4\frac{1}{2}$ sq. units

SECTION C (20 Marks)

Question 19

[3×2]

(a) A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of ₹ 8 per unit. Find the break-even point.

(b) The total cost function for a production is given by $C(x) = \frac{3}{4}x^2 - 7x + 27$.

Find the number of units produced for which M.C. = A.C.
(M.C.= Marginal Cost and A.C. = Average Cost.)

(c) If $\bar{x} = 18, \bar{y} = 100, \sigma_x = 14,\ \sigma_y = 20$ and correlation coefficient r_{xy} = 0·8, find the regression equation of y on x.

Comments of Examiners

(a) Many candidates misinterpreted the question and calculated incorrect revenue function, which led to the incorrect calculation of break-even point.

(b) This question was well attempted by most of the candidates. A few candidates made calculation errors in the process of simplification.

(c) Several candidates wrote incorrect formula for b_{yx} due to lack of understanding of regression coefficients and their formulae.

Suggestions for teachers

- Explain all technical terms and formulae of the chapter on cost function along with a good number of examples.
- Explain correct usage of formulae for regression coefficients b_{yx} and b_{xy} with the help of examples.
- Advise the students to maintain highest level of accuracy while solving problems from this topic.

MARKING SCHEME

Question 19

(a)	Given fixed cost = Rs 24000 Let number of units be: x Selling price per unit $p(x)$= Rs.8

	$\therefore$ Revenue function R(x) = 8x Cost function = C (x) = 24000 + 25% of 8x = 24000 + 2x For breakeven points: R(x) = C(x) $8x = 24000 + 2x$ $6x = 24000$ $x = 4000$ units
(b)	$C(x) = \frac{3}{4}x^2 - 7x + 27$ $A.C. = \frac{C(x)}{x} = \frac{3}{4}x - 7 + \frac{27}{x}$ $M.C. = \frac{d}{dx}(C(x)) = \frac{3}{4} \cdot 2x - 7$ $= \frac{3}{2}x - 7$ $\because$ AC = MC $\frac{3}{4}x - \cancel{7} + \frac{27}{x} = \frac{3}{2}x - \cancel{7}$ $3x^2 + 108 = 6x^2$ $3x^2 = 108$ $x^2 = 36$ $x = 6$
(c)	Given $\bar{x} = 18, \bar{y} = 100, \sigma_x = 14, \sigma_y = 20, r = 0.8$ $b_{yx} = r.\frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{20}{14} = \frac{8}{7}$ Regression equation y on x $y - \bar{y} = b_{yx} \cdot (x - \bar{x})$ $y - 100 = \frac{8}{7}(x - 18)$ $7y - 700 = 8x - 144$ $8x - 7y + 556 = 0$

Question 20

[4]

(a) The following results were obtained with respect to two variables x and y:

$$\sum x = 15, \sum y = 25, \sum xy = 83, \sum x^2 = 55, \sum y^2 = 135 \text{ and } n = 5$$

(i) Find the regression coefficient b_{xy}.

(ii) Find the regression equation of x on y.

OR

(b) Find the equation of the regression line of y on x, if the observations (x, y) are as follows:

(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)

Also, find the estimated value of y when x = 14.

Comments of Examiners

(a) Several candidates wrote incorrect formulae for regression coefficients b_{yx} and b_{xy} and thereby substituted incorrect values of regression coefficients in regression equations.

(b) Many candidates wrote incorrect formula for regression coefficients which caused error in finding the regression equation *y* on *x*. Several candidates were unable to interpret the result of *y* for *x*=14.

Suggestions for teachers

- Explain with examples, the correct usage of formulae for regression coefficients b_{yx} and b_{xy} , identification of regression equations, etc.
- Advise students to practice a variety of problems based on regression.

MARKING SCHEME

Question 20

(a)

$$\sum x = 15,\ \sum y = 25\ \sum xy = 83, \sum x^2 = 55,\ \sum y^2 = 135,\ n = 5$$

$$b_{xy} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}} = \frac{83 - \frac{15 \cdot \boxed{25}}{\cancel{5}}}{135 - \left(\frac{625}{5}\right)} = \frac{4}{5} = 0.8$$

$$\bar{x} = \frac{15}{5} = 3, \bar{y} = \frac{25}{5} = 5$$

Regression equation *x* on *y*

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 3 = \frac{4}{5}(y - 5)$$

$$5x - 15 = 4y - 20$$

$$5x - 4y + 5 = 0$$

OR

(b)

X	y	x^2	xy
1	4	1	4
2	8	4	16
3	2	9	6
4	12	16	48
5	10	25	50
6	14	36	84
7	16	49	112
8	6	64	48
9	18	81	162
45	90	285	530
$\sum x$	$\sum y$	$\sum x^2$	$\sum xy$

	$\bar{x} = \frac{45}{9} = 5, \bar{y} = \frac{90}{9} = 10$ $b_{yx} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{530 - \frac{45 \cdot 90}{9}}{285 - \frac{(45)^2}{9}} = \frac{4}{3}$ Regression equation *y* on *x*: $y - \bar{y} = byx(x - \bar{x})$ $y - 10 = \frac{4}{3}(x - 5)$ $3y - 30 = 4x - 20$ $4x - 3y + 10 = 0$ at $x = 14$ $56 - 3y + 10 = 0$ $3y = 66$ or $y = 22$

Question 21

[4]

(a) The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$ where x is the number of units produced. How many units should be produced to minimise the marginal cost?

OR

(b) The marginal cost function of x units of a product is given by MC = $3x^2 - 10x + 3$. The cost of producing one unit is ₹ 7. Find the total cost function and average cost function.

Comments of Examiners

(a) Many candidates minimised the cost function instead of the marginal cost function. Some candidates did not complete the second order derivative test for minimisation.

(b) Several candidates integrated MC(x) to find C(x) without adding constant to the C(x). This caused an incorrect solution.

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Suggestions for teachers

- Revise the concepts of application of derivatives regarding increasing/ decreasing functions and maxima/minima in the context of functions like Cost function, Marginal Cost and Average Cost, etc.
- Explain the application of integration in Commerce and Economics supported with an appropriate number of examples.
- Clarify to the students with the help of examples, the importance and significance of Constant in problems based on integration applications.

MARKING SCHEME

Question 21

(a)	$C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$ $MC(x) = x^2 - 90x - 900$ $\frac{d}{dx}(MC) = 2x - 90 = 0$ $x = 45$ $\frac{d^2}{dx^2}(MC) = 2 > 0$ at $x = 45$ MC is minimum. **OR**
(b)	$MC = 3x^2 - 10x + 3$ $C(x) = \int MC\ dx + c$ $C(x) = x^3 - 5x^2 + 3x + c$ Cost of producing 1 unit is Rs.7 $\therefore 7 = 1 - 5 + 3 + c$ $\Rightarrow$ c = 8 $\therefore$ Total cost function = $C(x) = x^3 - 5x^2 + 3x + 8$ Average cost function = $x^2 - 5x + 3 + \frac{8}{x}$

Question 22

[6]

A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income? Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph.

Comments of Examiners

Many candidates could not express the given constraints in the form of linear inequalities correctly. Several candidates did not write non-negative constraints x ≥ 0 and y ≥ 0. A few candidates did not indicate the feasible region on the graph correctly.

Suggestions for teachers

- Revise the concept of solving linear inequalities studied in the previous class.
- Explain the method of identifying the constraints and forming the corresponding linear inequalities and sketching the graph of the same to identify the feasible region for the objective function.
- Adequate practice in solving different types of Linear Programming problems must be given to students.

MARKING SCHEME

Question 22

Let the number of units of product A and product B to be made are x and y respectively.

Therefore, the problem can be written as:

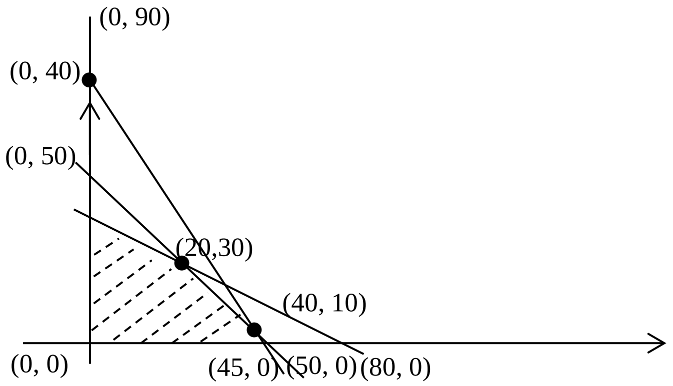
Product	No. of units	Wood used per unit			Income per unit
		Teak	Plywood	Rosewood	
A	x	2	1	1	48
B	y	1	2	1	40
Available stock		90	80	50	

Maximise $Z = 48x + 40y$ subject to the constraints

$2x + y \leq 90,\ x + 2y \leq 80,\ x + y \leq 50$

$x \geq 0,\ y \geq 0$

Its graphical solution is:



A at (0,0)	Z = 0
(45, 0)	Z = 48 x 45 + 40 x 0 = 2160
(40, 10)	Z = 48 x 40 + 40 x 10 = 2320
(20, 30)	Z = 48 x 20 + 40 x 30 = 2160
(0, 40)	Z = 48 x 0 + 40 x 40 = 1600

Hence, Z is maximum, 2320 at (40, 10)

$\therefore x = 40,\ y = 10$

Note: For questions having more than one correct answer/solution, alternate correct answers/solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS

Topics found difficult by candidates

- Increasing and decreasing functions.
- In proving the function “onto”.
- Applying formulae of inverse trigonometric functions.
- Finding the area of a triangle using determinant.
- Proving differentiability.
- Application of derivatives, in particular, rate measurer, Maxima and Minima.
- Conditional Probability and Bayes’ theorem.
- Family of planes and equation of straight line of 3D Geometry.
- Area under the curve.
- Application of calculus in Commerce and Economics.

Concepts in which candidates got confused

- Between the composite functions $f \circ g$ and $g \circ f$.
- Between open and closed interval in Mean Value theorem.
- Between mutually exclusive and independent events.
- Identifying the type of differential equation.
- Definite integrals and their properties.
- Between Marginal cost and average cost
- Between the regression coefficients b_{yx} and b_{xy} and regression lines y on x and x on y.
- Between Dot and Cross product of vectors.

Suggestions for candidates

- Avoid selective study. Study and practice the entire syllabus keeping in view the pattern of and the weightage of each chapter in the question paper.
- Revise the concepts of Class XI and integrate them with Class XII syllabus.
- Revise concepts like factorization, formulae of algebraic expressions, etc.
- Understand the concepts of each topic and practice adequate number of problems on a regular basis, taking the assistance of the teacher, wherever required.
- Make chapter/topic wise list of all formulae, comprehend and revise the same at regular intervals.
- Utilize the reading time of 15 minutes for reading the questions carefully and make a note of the various key points in the statements of the questions.
- Increase the level of accuracy to the highest levels while solving problems based on numerical values.
- Manage your time effectively while attempting the question paper. Practise mock/sample papers by strictly adhering to the stipulated time.