Electric Potential

Electric potential at a point is defined as the work done by an external agent in carrying a unit positive test charge from infinity to a point in the electric field

$$V = \frac{W}{q_0}$$

SI unit of potential is volt

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ J/C}$$

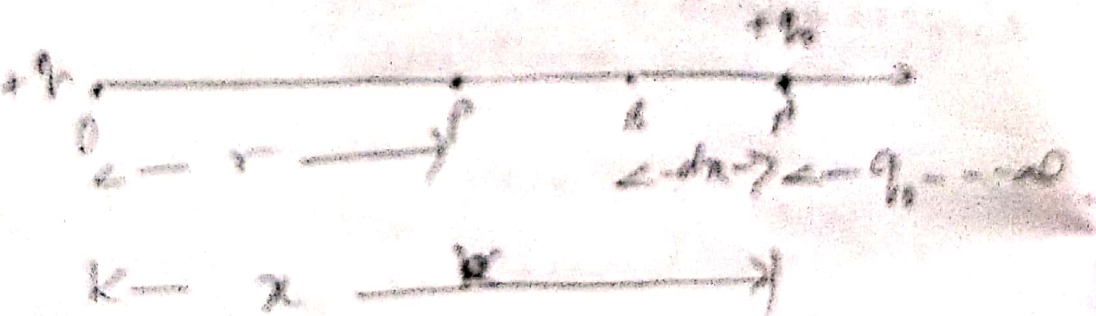
Dimensional formula = $[ML^2T^{-3}A^{-1}]$

Potential difference

$$V_A - V_B = \frac{W}{q_0}$$

1 electron volt = 1.6×10^{-19} joule

Electric potential due to a point charge



Let a test charge $+q_0$ be placed at point A, distance x from O. By Coulomb's law, electric force

$$F = \frac{1}{4\pi\varepsilon_0 k} \frac{q q_0}{x^2}$$

Work done, $dW = F(-dx)$

Total work done is

$$W = -\int_{\infty}^{r} F\, dx$$

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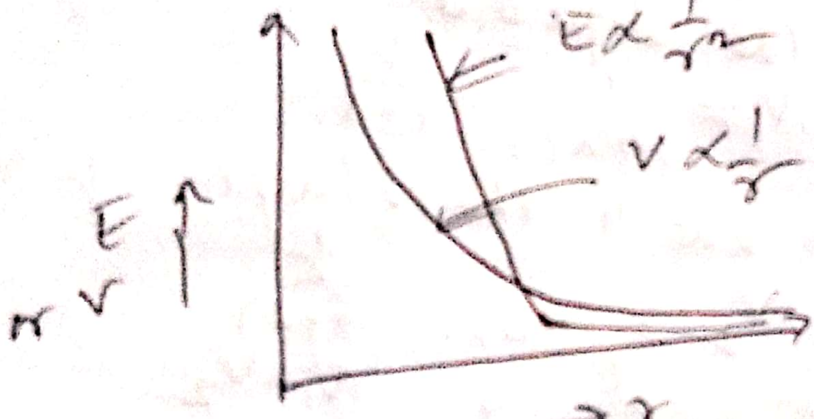
$$= -\frac{q q_0}{4\pi\epsilon_0 k}\int_{\infty}^{r} \frac{1}{x^2}\,dx$$

$$= -\frac{q q_0}{4\pi\epsilon_0 k}\left[-\frac{1}{x}\right]_{\infty}^{r} = \frac{-q q_0}{4\pi\epsilon_0 k}\left[-\frac{1}{r}+\frac{1}{\infty}\right]$$

$$W = \frac{1}{4\pi\epsilon_0 k}\frac{q q_0}{r}$$

Potential $V = \frac{W}{q_0} = \frac{1}{4\pi\epsilon_0 k}\frac{q}{r}$

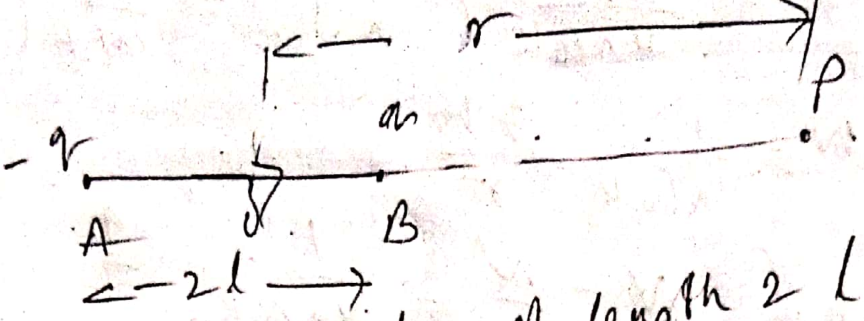


Potential due to GROUP of point charges

Let the point charges $+q_1$, $+q_2$, $-q_3$ and $-q_4$ at a distances r_1, r_2, r_3 and r_4. The resultant potential

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} - \frac{q_3}{r_3} - \frac{q_4}{4}\right]$$

(i) Electric potential at a point on the axis of the dipole.



AB is a dipole, of length 2l. The potential at P due to charge +q of the dipole is

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r-l}$$

and that due to the charge -q is

$$V_2 = -\frac{1}{4\pi\epsilon_0}\frac{q}{(r+l)}$$

Total potential $V = V_1 + V_2$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r-l} - \frac{1}{r+l}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{(r+l)-(r-l)}{r^2-l^2}\right]$$

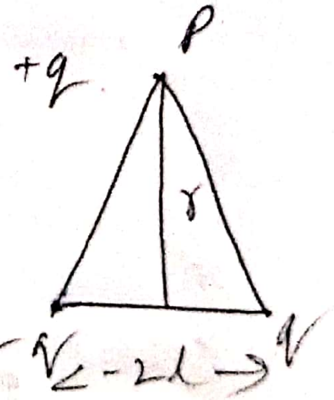
$$= \frac{q}{4\pi\epsilon_0}\frac{2l}{(r^2-l^2)} = \frac{1}{4\pi\epsilon_0}\frac{2ql}{r^2-l^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{P}{r^2-l^2}$$

If $r \gg 2l$

$$\therefore V = \frac{1}{4\pi\epsilon_0}\frac{P}{r^2} \text{ volt}$$

(ii) Potential at a point on the Equatorial line of the dipole



Potential at P due to the charge +q

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{BP}$$

and that due to charge '-q' is

$$V_2 = -\frac{1}{4\pi\epsilon_0}\frac{q}{AB}$$

Resultant potential at P is

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{BP} - \frac{1}{AP}\right) = 0 \quad (BP = AP)$$

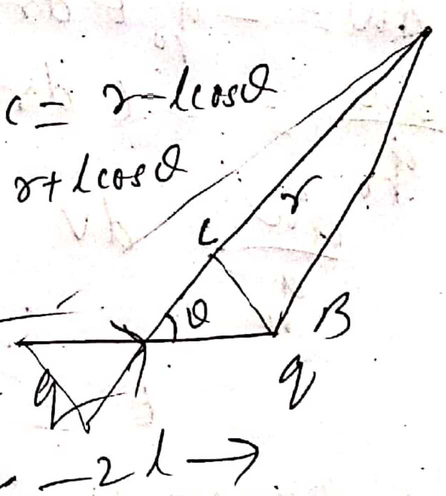
Potential at any point

Let P be any ~~distance~~ point at a distance r from O.

Since $r >> l$

$BP = CP = OP - OC = r - l\cos\theta$

$AP = DP = OP + OD = r + l\cos\theta$



Potential at P due to +q of the dipole is

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{BP}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(r - l\cos\theta)}$$

due to −q charge

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r + l\cos\theta)}$$

Total potential is $V = V_1 + V_2$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r + l\cos\theta - r + l\cos\theta}{(r - l\cos\theta)(r + l\cos\theta)}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q \cdot 2l\cos\theta}{(r^2 - l^2\cos^2\theta)} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{(r^2 - l^2\cos^2\theta)}$$

If $r >> l$ then

$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$

In vector form $V = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$